THE EFFECT OF HEAT TRANSFER ON UNSTEADY PERIODIC MOTION OF A GAS IN TUBES

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The one-dimensional unsteady motion of a gas in a straight tube is described with the aid of the equations of continuity, of motion, of state, and of energy; in addition, friction and thermal flow are assumed to be quasistationary. Solution of the linearized equations shows that when heat transfer is taken into account, "entropic" waves of pressure and velocity appear in the gas. As the result of heat transfer changes also occur in the propagation constants for ordinary waves; there is, compared with an adiabatic process, an increase in the distribution of friction.

In describing the unsteady motion of a gas in a horizontal tube of constant cross section it is customary to use the system of equations of continuity, of motion, and of state [1]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho W) = 0 \tag{1}$$

$$\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + F = 0$$

$$p = \rho g R^\circ T$$

Here x is a coordinate whose direction coincides with that of the mean flow velocity; t is the time; p(x, t), W(x, t), ρ(x, t), and T(x, t) are, respectively, the pressure, velocity, density, and temperature of the gas, averaged over a given cross section; moreover, the velocity of the gas is assumed to be significantly less than the velocity of sound; the quantity F, defined below, is associated with friction.

To obtain the pressure and velocity it is necessary to supplement these equations with a relation which takes into account the heat transfer between the gas and the surrounding medium. If it be assumed as in [1] that the propagation of waves of pressure and velocity of the gas is a process which takes place isothermally ($T = T_0$), we obtain the equation $p = c_0^2\rho$, where the speed of sound $c_0 = \sqrt{gR^\circ T_0}$, so that the actual number of unknowns is diminished by two. If it be assumed that the wave propagation process is adiabatic [2], then the number of variables is also decreased, since, in this case, with linearization, we have the valid relation $p_1 = c_0^2\rho_1$, where $c_0 = \sqrt{kgR^\circ T_0}$ is the speed of sound and T_0 is the stationary component of the gas temperature, p_1 and ρ_1 being the nonstationary components of the pressure and density of the gas.

By taking into account in the initial equations the process of heat transfer between a gas and its surrounding medium, we are able, first of all, to solve the problem of when such assumptions are justifiable and, secondly, we can calculate the gas dynamic processes in portions of tubular systems where the heat transfer is intense (condensers, heat exchangers). The difficulties which arise in solving the latter problem compel us to assume that the system is frictionless (F = 0); in [3] an approximate method was worked out in this case for solving the wave equation with the velocity of sound variable with distance, the method being based on the use of integral characteristics. The same problem was handled in [4] by a computational method using nonhomogeneous electrical lines, and in [5] there were deduced ordinary differential equations, solvable in terms of special functions, to which the wave equation reduces when the mean gas temperature varies with length according to various laws. With reference to the latter papers it should be

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noted that the main reason for variation of the gas temperature was considered to be the presence of heat transfer between the tube and the surrounding medium; in addition, the authors, without analyzing the heat transfer process, assumed the law of variation of the mean gas temperature with length to be given. In these papers no account is taken of the heat transfer between the gas and the tube walls, thus substantially affecting the process of the propagation of pressure and velocity waves in the gas.

To solve the problem of estimating the influence of heat transfer on gasdynamic processes in pipeline systems it is necessary to adjoin to Eqs. (1) the energy equation

$$\rho Q+\frac{\partial}{\partial t}\left[\rho\left(c_v T+\frac{W^2}{2g}\right)\right]+\frac{\partial}{\partial x}\left[\rho W\left(c_p T+\frac{W^2}{2g}\right)\right]=0 \tag{2}$$

The main difficulties in the analysis of Eqs. (1) and (2) are connected with the determination of the magnitudes of the thermal flow Q and the friction F for a nonstationary regime. We shall assume that the frictional process is quasistationary [1]; then for the turbulent motion of piston compressors, which is characteristic of pipeline systems, we have, for the Reynolds number range $R = 10^5$-10^6

$$F=\xi W^2 \qquad (\xi = {}^1/_2\ \lambda\ /\ d) \tag{3}$$

where λ is the coefficient in the Darcy–Weisbach formula, and d is the tube diameter. In determining the thermal flow Q we shall also consider the heat transfer process to be quasistationary; such an approach is justified because, for $\omega = 10$–$100\ \sec^{-1}$ the period of the oscillations of the nonstationary component of the gas velocity turns out to be 2-3 orders larger than the time required to establish the temperature in the laminar sublayer. If we take the temperatures in the resulting thermal flow to be the instantaneous temperature of the gas T(x, t) and the external temperature, considerable complexities arise since the heat transfer coefficient itself depends on the difference of the temperatures [6].

A more suitable choice of defining temperatures is the instantaneous gas temperature T(x, t) and the wall temperature $T_W(x)$, moreover it may be assumed that as a consequence of the high oscillational frequency the nonstationary component of the gas temperature has no influence on the wall temperature, so that the latter is determined by the static temperature of the gas at the tube entrance and by the external heat exchange. We remark that for relatively short portions of pipeline systems, which do not pass through heat exchangers, the mean gas temperature is practically constant with length since the external heat exchange is very insignificant. We shall consider the portions of pipelines passing through compressors and heat exchangers to be split up into portions for which the mean gas temperature on each portion is constant. Then for both cases

$$T_w(x)=T_0, \quad Q=\alpha\left[T(x,t)-T_0\right], \quad \alpha=\xi c_p P^{-2/3} W \tag{4}$$

Here α is the heat emission coefficient for forced turbulent motion of the gas in the tube, for which, in a given case, Colbourn's formula is applicable [6]; molecular heat conduction is neglected.

We note that the energy equation (2) can, with the aid of the system (1), be rewritten in one of the following forms:

$$\frac{D}{dt}\left[c_v T+\frac{W^2}{2g}\right]+R^\circ W\frac{\partial T}{\partial x}-\frac{R^\circ T}{\rho}\frac{\partial\rho}{\partial t}+Q=0 \qquad \left(\frac{D}{dt}=\frac{\partial}{\partial t}+W\frac{\partial}{\partial x}\right) \tag{5}$$

$$\frac{\partial p}{\partial t}+W\frac{\partial p}{\partial x}-\frac{kp}{\rho}\left[\frac{\partial\rho}{\partial t}+W\frac{\partial\rho}{\partial x}\right]=\rho(k-1)(WF-gQ)\left(k=\frac{c_p}{c_v}\right) \tag{6}$$

If we introduce the entropy $S = c_v \ln p - c_p \ln \rho$, then from Eqs. (6) we have

$$\frac{DS}{dt}=\frac{1}{T}\left(\frac{WF}{g}-Q\right) \tag{7}$$

If in Eqs. (1) and (7) we pute F = 0, Q = 0, we obtain the case treated in [7]; we show below that, in addition, there is variation in comparison with an isentropic process.

We linearize the system (1), (6) by decomposing the quantities p, W, ρ, and T into static and small dynamic components:

$$p=p_0+p_1, \quad W=W_0+W_1, \quad \rho=\rho_0+\rho_1, \quad T=T_0+T_1$$

We obtain

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial W_1}{\partial x} + W_0 \frac{\partial \rho_1}{\partial x} = 0 \tag{8}$$

$$\rho_0 \frac{\partial W_1}{\partial t} + \rho_0 W_0 \frac{\partial W_1}{\partial x} + \frac{\partial p_1}{\partial x} + 2\xi\rho_0 W_0 W_1 + \xi W_0^2 \rho_1 = 0$$

$$p_1 - gR^\circ \rho_0 T_1 - gR^\circ T_0 \rho_1 = 0$$

$$\rho_0 \frac{\partial p_1}{\partial t} + \rho_0 W_0 \frac{\partial p_1}{\partial x} - kp_0 \frac{\partial \rho_1}{\partial t} - kp_0 W_0 \frac{\partial \rho_1}{\partial x} - 3(k-1)\xi\rho_0^2 W_0^2 W_1$$
$$-2(k-1)\xi\rho_0 W_0^3 \rho_1 + kgR^\circ \xi P^{-2/3} \rho_0^2 W_0 T_1 = 0$$

Considering only forced oscillations (experimental studies, see [2], have shown that the characteristic oscillations in pipeline systems of piston compressors are damped rapidly and have practically no effect on the gasdynamic process) we seek a solution of the system (8) in the form

$$p_1(x,t) = p_1(x) e^{j\omega t}, \quad W_1(x,t) = W_1(x) e^{j\omega t}, \quad \rho_1(x,t) = \rho_1(x) e^{j\omega t}$$
$$T_1(x,t) = T_1(x) e^{j\omega t}$$

If we eliminate the temperature $T_1(x)$ from these equations we obtain a system of ordinary differential equations of the first order; the corresponding characteristic equation has the form

$$\det |a_{\lambda\mu}| = 0 \qquad (\lambda, \mu = 1, 2, 3) \tag{9}$$

where

$$a_{11} = -c_0^2 (j\omega + W_0\gamma) - 2(k-1)\xi W_0^2 - \xi P^{-2/3} W_0 c_0^2$$
$$a_{12} = j\omega + W_0\gamma + k\xi P^{-2/3} W_0 \; a_{13} = -3(k-1)\xi P_0 W_0^2, \; a_{21} = j\omega + W_0\gamma$$
$$a_{22} = 0, \quad a_{23} = \rho_0\gamma, \quad a_{31} = \xi W_0^2, \quad a_{32} = \gamma$$
$$a_{33} = \rho_0 (j\omega + W_0\gamma + 2\xi W_0) \qquad (c_0 = \sqrt{kgR^\circ T_0})$$

For the case $\xi = 0$ the roots of this equation are easily found:

$$\gamma_1 = -\frac{j\omega}{W_0}, \qquad \gamma_2 = -\frac{j\omega}{c_0 + W_0}, \qquad \gamma_3 = \frac{j\omega}{c_0 - W_0} \tag{10}$$

The presence of three characteristic roots testifies to the fact that rejection of isentropicity leads to a third phenomenon, an "entropic wave," which, as is evident from the matrix (9) for $\xi = 0$ and $\gamma = -j\omega/W_0$ effects only the temperature and density of the gas; the values of $p_1(x, t)$ and $W_1(x, t)$ remain the same as for an isentropic process. To find the propagation constants γ for the general case ($\xi \neq 0$) we discard, in the coefficients of the equation in γ obtained from Eq. (9), terms of order $(W_0/c_0)^2$ (for gas velocities encountered in practice in pipeline systems of piston compressors this quantity is of order 10^{-2}). If we select the roots (10) as the zeroth approximation, we find the first approximation for the roots of the resulting equation by Newton's method. With a precision sufficient for calculations of pipeline systems, the results have the form

$$\gamma_1 = \frac{-j\omega}{W_0} - \xi P^{-2/3}, \quad \gamma_{2,3} = \frac{\mp j\omega}{c_0 \pm W_0} \mp \frac{\xi W_0}{c_0}\left[1 + \frac{k-1}{2} P^{-2/3}\right] \tag{11}$$

We note that independently of the values found it follows from the form of the matrix (9) that: if γ is a root of Eq. (9), then in a single row no two of the three coefficients can vanish; this means that the presence of friction leads to the appearance of entropy waves of pressure and velocity of the gas. In addition, taking heat exchange into account also leads to a change in the propagation constants for ordinary waves: from the expressions for γ_2 and γ_3 it is evident that in comparison with the adiabatic case, to within the limits of precision chosen, the imaginary part of the constant diffusion does not vary, but there is, in fact, a 25% growth in the distributed friction for k = 1.4, P = 0.7. The solution of the system (8), using the same assumptions as before, may be written in the form

$$\rho_1 = [A_1 e^{\gamma_1 x} + A_2 e^{\gamma_2 x} + A_3 e^{\gamma_3 x}] e^{j\omega t} \tag{12}$$

$$T_1 = \frac{T_0}{\rho_0}[-A_1 e^{\gamma_1 x} + (k-1) A_2 e^{\gamma_2 x} + (k-1) A_3 e^{\gamma_3 x}] e^{j\omega t} \tag{13}$$

$$p_1 = c_0^2 \left[-\frac{j\xi W_0^3}{\omega c_0^2} A_1 e^{\gamma_1 x} + A_2 e^{\gamma_2 x} + A_3 e^{\gamma_3 x}\right] e^{j\omega t} \tag{14}$$

$$W_1 = \frac{c_0}{\rho_0}\left[\frac{j\xi W_0}{c_0} P^{-2/3} A_1 e^{\gamma_1 x} + A_2 e^{\gamma_2 x} - A_3 e^{\gamma_3 x}\right] e^{j\omega t} \tag{15}$$

A peculiarity of entropy waves in comparison with ordinary waves consists in the fact that they decay much more rapidly. In fact, the ratio of the amplitudes of the oscillations at two points, separated from one another by a distance l, for entropy and ordinary waves, is, respectively, $\exp \mathrm{Re}(\gamma_1 l)$ and $\exp \mathrm{Re}(\gamma_2 l)$. From this, taking the relations (11) and (3) into account, it follows that for an ordinary wave and an entropy wave to undergo the same decrease in amplitude, the ratio l/d in the first case must be at least ten times larger than in the second. Or, to put it differently, for $l/d > 80$ ($\lambda = 0.04$) the amplitude of an entropy wave has decreased tenfold. Thus, if the initial amplitude of an entropy wave is not too large [we note that in relations (14) and (15) the coefficients are of order 10^{-1}–10^{-2} in absolute value) its influence manifests itself only over a short entrance portion of the tube.

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